



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

beginning of the term as the epoch of reference. The required rate is then that at which the present value of the annuity shall be equal to the sum borrowed.

We thus have to satisfy the equation

$$V=S,$$

where V is the present value of the annuity expressed in terms of i , the required rate per £1.

By the general formula for the value of an annuity whose payments are b_1, b_2, b_3 , &c., adapted to this case, we have

$$V = \frac{b_1}{i} + \frac{\Delta^1 b_1}{i^2} + \frac{\Delta^2 b_1}{i^3} - v^n \left(\frac{b_{n+1}}{i} + \frac{\Delta^1 b_{n+1}}{i^2} + \frac{\Delta^2 b_{n+1}}{i^3} \right);$$

and we have to find the value of i which makes this expression equal to S , the sum borrowed.

We easily find for the $(n+1)$ th payment:—

$$b_{n+1} = \{S - an[m + \frac{1}{2}(n-1)d]\}i + a(m + nd)(1+p);$$

differencing twice with respect to n , we get,

$$\begin{aligned}\Delta^1 b_{n+1} &= a[d(1+p) - (m + nd)i], \\ \Delta^2 b_{n+1} &= -adi, \text{ constant.}\end{aligned}$$

And if we make in these $n=0$, we get,

$$\begin{aligned}b_1 &= Si + am(1+p), \\ \Delta^1 b_1 &= a[d(1+p) - mi], \\ \Delta^2 b_1 &= -adi.\end{aligned}$$

These expressions may be verified on M. Violeine's problem; but they are equally applicable to any problem of the same kind. By the use of them the preliminary work on p. 94 is saved.

HOME AND FOREIGN INTELLIGENCE.

COMMERCIAL UNION ASSURANCE COMPANY.

Established 1862.

REPORT TO THE DIRECTORS BY THE ACTUARY, MARCH, 1868.

PART I.

Detailed information upon the Business and constitution of the Office.

(1.) *Business and Regulations.*

The first Policy was granted by the Life Branch on the 1st day of May, 1862.

The number and amount of the several kinds of Policies granted by the Company, of those under which its liability has run-off, and of those remaining in force on the 31st December last (which are the subject of the present valuation), are given in the following Tabular statements.

TABLE OF POLICIES GRANTED.

Description.	No.	Amount.
		£
With Participation.		
For the Whole Term of One Life . .	1,769	1,268,000
For various Terms and Contingencies .	89	73,750
Without Participation.		
For the Whole Term of One Life . .	310	345,070
For various Terms and Contingencies .	271	366,390
	2,439	2,053,210

TABLE OF POLICIES VOID.

Description.	No.	Amount.
		£
With Participation.		
For the Whole Term of One Life . .	300	179,560
For various Terms and Contingencies .	11	6,700
Without Participation.		
For the Whole Term of One Life . .	61	88,250
For various Terms and Contingencies .	97	135,792
	469	410,302

TABLE OF POLICIES IN FORCE.

AT 31ST DECEMBER, 1867.

Description.	No.	Amount.
		£
With Participation.		
For the Whole Term of One Life . .	1,469	1,088,440
For various Terms and Contingencies .	78	67,050
Without Participation.		
For the Whole Term of One Life . .	249	256,820
For various Terms and Contingencies .	174	230,598
	1,970	1,642,908

(2.) *Funds and Interest.*

The Funds of the Life Branch, after crediting the account with the Premiums which were due but had not been paid owing to the Days of

Grace not having expired, and also with the Interest which had accrued on the investments up to the 31st December, amounted on that day to £173,501. The following Table shows the progress in the accumulations of the

LIFE BRANCH.

Amount of the Funds on the 31st December, 1862,	£7,732
Ditto " " " 1863	24,078
Ditto " " " 1864	52,993
Ditto " " " 1865	88,275
Ditto " " " 1866	124,896
Ditto " " " 1867	169,623

NOTE.—In the last amount, the Premiums due and Interest accrued are not included, as these items are omitted in the sums for the corresponding dates of the previous years.

The rate of Interest realised upon the Life Funds, throughout the whole period, has averaged nearly £5 per cent., and the investments on the 31st December were such as to yield £4 16s. 6d. per cent. As the rate of Interest at which the Premiums are expected to improve has been assumed in the valuation at 3 per cent. only, it will be seen that a very large margin of future profit has been provided in the item of Interest alone, though in future it can scarcely be expected that the present average rate will be maintained.

(3.) *Mortality.*

It has been a fact long known to Actuaries that the mortality of recently-selected lives falls considerably below that of any tabular rate. The Life Premiums of the Commercial Union are based upon the Carlisle Table of Mortality, and the results experienced by the Office, as compared with the mortality expressed by that Table, are as follows:—

TABLE.

Year.	Average Amount at Risk.	Expected Claims by the Carlisle Table.	Actual Claims.
1862	69,900	873	Nil.
1863	323,280	4,024	2,700
1864	640,942	8,227	1,300
1865	926,396	12,157	1,200
1866	1,119,179	14,940	3,600
1867	1,249,830	17,197	12,700*
Totals..	£4,329,527	£57,418	£21,500

* Of this amount £5,000 was re-assured.

PART II.

Valuation and Bonus.(1.) *Rate of Mortality Assumed.*

The Carlisle Table was selected as the basis upon which the Company's Premiums should be calculated, not because it was deduced from an exten-

sive collection of facts, nor because the rate of mortality furnished by it would be likely to represent that of Assured Life generally, but principally because it yielded a fair average of results, and possessed the incidental advantage of having in connection with it a large number of auxiliary tables useful for the calculation of Premiums on Special Assurances.

It will be seen from the following Table that the expectation of life, according to the Carlisle for persons under 40 years of age, has been confirmed by Mr Morgan, from the experience of the Equitable Society, and by Mr. Downes from that of the Economic; but the observations of these gentlemen and the Tables of the Combined Experience, the Amicable Society, and the Eagle Office—all Tables of authority—indicate that the Carlisle gives too favourable an estimate of the duration of life for persons above the age of 40.

TABLE.

MEAN DURATION OF LIFE.						
Age.	Carlisle Table.	ASSURED LIFE.				
		Equitable (Morgan).	Economic.	Experience of 17 Offices.	Amicable, 1851.	Eagle.
20	41·46	41·67	41·40	41·49	(Not given)	37·99
25	37·86	38·12	38·57	37·98	38·86	34·98
30	34·34	34·53	34·82	34·43	34·84	31·89
40	27·61	27·40	27·20	27·28	27·11	25·54
50	21·11	20·36	19·96	20·18	19·83	19·37
60	14·34	13·91	13·83	13·77	13·32	13·52

When recommending the adoption of the Carlisle Table as the basis for the calculation of the Premiums, I was influenced also in some measure by the experience of other Offices that a large number of the Policies effected would remain on the books for a few years only, and that the mortality of the Assured would for some years after selection be considerably below the rate furnished by the Carlisle or any other Table.

But if it be assumed that the Carlisle Table represents the average rate of mortality under Policies throughout their currency, and if the actual rate immediately after the Assurances are effected is considerably below the tabular one, it follows that the mortality, after the effects of selection have disappeared, will exceed that furnished by the Carlisle, in order to make the Table true for the whole duration of the Policies. I decided, therefore, to recommend you to authorise for the purpose of the valuation the adoption of a different rate of mortality from the Carlisle, * * * a Table of Mortality which required the greatest Reserve, and guaranteed, in the highest degree, the safety and future prosperity of the office.*

* The Carlisle Table, I am aware, has been very widely adopted by Companies in their valuations; and I observe that Mr. Thompson, the Actuary of the Standard Life Office, has recently stated that his Company had adhered to this Table for 30 years, and that the leading Offices had, one by one, been led to adopt it. I venture, however, to predict that, one by one, those Offices which have accepted it as the basis of their calculations will reject it (as an *unsafe* Table) when they find the new business in smaller proportion to the existing business at future valuations.

(2.) *Rate of Interest Assumed.*

The rate of Interest assumed in the calculation of the Premiums was 3 per cent. only, and as the same assumption has been made in the valuation, new Assurances will be placed under no disadvantage compared with existing Policies, as the liability under the latter will have been amply provided for. I have already stated that the average rate of Interest obtained by the Life Branch upon the monies invested is £4 16s. 6d. per cent. per annum.

It is not uncommon for Offices, in their valuations, to assume Interest at the rate of $3\frac{1}{2}$ per cent., and, in some cases, even 4 per cent. has been assumed, though the calculation of the Premiums was based upon 3 per cent.; but it is obvious that the adoption of a higher rate in the valuation indicates a diminution in a Company's resources for meeting outstanding obligations, as, of course, a much smaller Reserve is thus provided than would have been provided had a lower rate of Interest been assumed.

The following example may be adduced in illustration:—

Several Offices, by adopting a 3 per cent. Valuation Table, reserve for their liabilities from £2,000,000 to £5,000,000. Now, if the rate were assumed at 4 per cent., the Bonus immediately following the valuation would certainly be augmented by 10 per cent. of this Reserve, or from £200,000 to £500,000; but, as a consequence, the Bonuses at subsequent divisions would be diminished, if they did not entirely disappear.

(3.) *Margin for Expenses of Management, and for forming a Bonus Fund.*

In the present valuation the whole of the "loading" has been reserved, and only that part of the Premium which provides for the risk has been valued as an Asset.

(4.) *The Surplus ascertained, and Profits to be declared.*

The Tables used by Actuaries for the valuation of the Capital sums assured, assume that they will be paid at the end of the year in which the Lives fail, or, on an average, six months after death. As, however, most Offices agree to pay the amount assured three months after proof of death, the Tabular calculations yield a lower value than the true one, and, consequently, require adjustment. In this Office a greater adjustment than ordinary is necessary, on account of the Company's Claims being payable one month after death has been proved. The amount of this adjustment, or sum to be added to the net liability, is £6,279; and a further adjustment is requisite to meet the cases where an excess of Premiums is paid in the second half of each year instead of the payments being equally distributed throughout the year, as the Tabular calculations assume. These two adjustments amount together to £9,756, and being added to the net liability under Policies and Annuities produce the sum of £129,694.

The Balance-Sheet shows that the Assets of the Life

Branch, on the 31st December, amounted to .	£173,501	0	0
Deducting the total liability above-stated of .	129,694	0	0
There remains . . .	£43,807	0	0

as a divisible surplus, after assuming the Combined Experience Mortality with Interest at 3 per cent., and making the most ample provision for all

special contracts and Annuities.* Of this surplus I recommend that £41,000 be divided, leaving an additional small sum to be added to the large Reserve already provided.

One-fifth of this £41,000, or £8,200, belongs, according to the Deed of Settlement, to the Shareholders, and four-fifths, or £32,800, will be appropriated to the Assurances at present in force. This £32,800, if applied as a uniform Reversionary Bonus to all the Policies, will, on an average, yield a Bonus of rather more than £2 2s. per cent. per annum on the sum assured from their commencement, equivalent to something more than 65 per cent. of the Premiums paid.

The Surplus stated above leaves untouched a "loading" of the Premiums of £7,631 per annum, the present value of which is £118,936, and this may be taken as the measure of the provision made for future expenses and Bonus, exclusive, however, of the profit to be derived from a higher rate of Interest on the accumulated Premium Fund than the 3 per cent. assumed.

In the Valuation Summary, given in the Appendix, will be found full particulars of the value of each class of Policies, but the results may be conveniently summarised in the following form:—

SUMMARY VALUATION TABLE.

	No. of Policies.	Value of Sums Assured.	Value of Net Premiums Payable.	Difference, being Liability.	Value of Margin.
1. Assurances.		£	£	£	£
Participating Policies	1,547	548,279	478,223	70,056	118,588
Non-Participating „	368	183,353	159,047	24,306	16,890
Special „	55	..	..	36,772	..
	1,970	731,632	637,270	131,134	135,478
<i>Deduct Re-Assurances</i>	..	103,765	85,482	18,283	16,541
		627,867	551,788	112,851	118,937
2. Annuities.					
Present and Deferred	10	..	..	7,087	..
<i>Adjustments . .</i>	..	..	..	9,756	..
	..	627,867	551,788	129,694	118,937

(5.) *Distribution of the Surplus among the Policy-holders.*

The methods employed by Life Offices in the distribution of Profits are as remarkable for their number as for their diversity.

The only principle which appears to me equitable in the distribution of the Surplus is that which makes an allotment in proportion to the contribution of each Policy to the Profit Fund, and upon this principle the Profits ascertained by the present valuation will be apportioned to the Policy-holders.

* The Annuities, it should be observed, have been calculated by the Office Tables—that is to say, the Reserve made for those outstanding is the same as that at which the Office would be prepared to grant Annuities of the same amount at the advanced ages of the Annuitants.

(6.) *Modes in which the Assured may receive their Bonus.*

It appears desirable that, in distributing the Surplus to the Assured, their circumstances or convenience should be consulted, as much as possible, by the offer of several options of receiving their share, and it is, therefore, proposed that the Bonus shall be declared in the three following *reversionary* methods:—

- (a.) an immediate addition to the sum assured, to be payable with it at death.
- (b.) the addition of a very large Bonus to the sum assured, to be payable with it at death, provided the Assured have then attained the average expectation of life of persons of his age at entry.
- (c.) the application of the Bonus in making the sum assured payable like an ordinary Endowment Policy in the life-time of the Assured;

or that Policy-holders shall have the right to surrender their Reversionary Bonuses for

- (d.) a cash payment; or
- (e.) a permanent reduction of the Annual Premiums.

The election of one of the five preceding methods of receiving the Bonus, must be made before or on the 1st July next. It should be understood that if a Policy-holder decide to receive a cash payment or have his Premium permanently reduced, the Bonus allotted to him at the next division of profits will be declared upon the original amount only of his Policy; but that if he accept any one of the three reversionary methods he will, at the second distribution, receive a larger Bonus than if he had selected a cash payment or reduction of premium, as the Bonus will be calculated upon the original amount of the Policy and the sum by which that Policy has been augmented at the present division.

TABLE.

Showing the approximate amount of Bonus upon a Policy for £1,000 effected in the year 1863.

Age at Entry.	Present Age.	REVERSIONARY BONUS.			
		Immediate.	Deferred.		
			Age.	Amount.*	
		£ s. d.		£ s. d.	
20	25	104 0 0	61	272 0 0	
30	35	104 10 0	64	259 0 0	
40	45	105 0 0	68	257 10 0	
50	55	110 0 0	71	253 0 0	
60	65	121 10 0	74	235 0 0	

* Payable with the sum assured, provided the Assured attain the average expectation of life of persons of his age at entry. By this plan a large addition to the Policy is made at each successive division of profits.

Summary and Valuation of the Policies of the Commercial Union Assurance Company, as at 31st December, 1867.

Description of Transactions.	No. of Policies.	Sums Assured.	Office Yearly Premiums	Net Yearly Premiums.	Net Liability.*
Assurances.					
1. WITH PARTICIPATION.					
For the Whole Term of Life	1,469	£ 1,088,440	£ 34,657	£ 27,900	£ 65,082
Do. (Premiums by a limited number of payments)	13	12,550	963	805	2,491
Do. (Increasing Premiums)	7	5,800	137	107	141
Endowment Assurances	25	8,000	361	290	491
Endowments	1	2,000	69	58	413
Deferred	15	31,500	688	535	1,069
On Two or more Lives	17	7,200	255	194	369
Assurances with Profits	1,547	1,155,490	37,130	29,889	70,056
2. WITHOUT PARTICIPATION.					
For the Whole Term of Life	249	256,820	9,481	8,593	20,136
Do. (Premiums by a limited number of payments)	1	800	19	17	89
Do. (Increasing Premiums)	50	43,210	828	718	933
Endowment Assurances	32	25,893	2,148	1,980	2,177
Joint Lives	15	7,450	301	261	281
On the Longest of Two or more Lives	10	7,570	150	130	382
Deferred	1	1,000	9	7	26
Children's Endowments	10	1,384	45	42	282
Policies not comprised in the above Class (Including Assurances for Terms of Years, against the Contingency of Issue, Contingent Survivorships, Endowments at short dates, including one for a large amount due in 1868, &c. &c.)	55	143,291	3,950	3,555	35,458
Extra Premiums payable	..	..	951	..	1,315
Assurances without Profits	423	487,418	17,882	15,303	61,079
Total Assurances	1,970	1,642,908	55,012	45,192	131,135
Deduct Re-Assurances	117	222,874	8,302	7,064	18,283
Net Amount of Assurances	..	1,420,034	..	..	112,852
Adjustments, to provide for the payment of Policies one month after proof of death	..	..	..	..	6,279
Do. to equalise the excess of Premiums payable in the second half of any year	..	..	..	..	3,477
Annuities.					
Present and Deferred	10	per annum 906	..	..	7,087
Total of the Results	..	..	46,710	38,128	129,695

* Value by the Combined Experience Tables, Interest 3 per Cent.

1868.]

Commercial Union Assurance Company.

407

Consolidated Account of the Receipts and Expenditure of the Life Branch, from 1st May, 1862, to 31st December, 1867.

1867.	£	s.	d.	£	s.	d.
Dec. 31. To New Premiums, including Single Premiums, and Premiums by a limited number of						
Payments	74,279	16	8			
„ Renewal Premiums	148,352	0	9			
				222,631	17	5
„ Consideration for Annuities				7,548	18	10
„ Bonus on Re-Assurances				42	1	0
„ Fines for Extension of Time				165	4	4
„ Interest				15,748	9	1
				£246,136	10	8
1867.				£	s.	d.
Dec. 31. By Claims				11,300	0	0
„ Surrenders				2,103	10	0
„ Re-Assurances				36,714	16	9
„ Annuities				3,575	8	9
„ Commission, less that received on Re-Assurances				8,243	1	7
„ Expenses of Management				14,576	5	0
Balance				169,623	8	7
				£246,136	10	8

Balance Sheet of the Life Branch of the Commercial Union Assurance Company on 31st December, 1867.

LIABILITIES.

I. AMOUNTS DUE BY THE LIFE BRANCH—			
1. Claims by Death, not yet payable	£5,100	0	0
2. Commission due to Agents	1,227	13	2
3. Re-Assurance Premiums	1,102	18	4
II. VALUE OF LIABILITIES UNDER ASSURANCE AND ANNUITY			
POLICIES, per Summary Valuation Statement	129,694	0	0
BALANCE	43,807	15	0
	£180,932	6	6

ASSETS.

Loans on Mortgages	£116,311	0	4
„ „ Policies	5,475	3	1
Great Southern of India Railway Guaranteed Stock	6,026	5	7
£4,600 Canada 5 per Cent. Bonds	4,134	5	0
Miscellaneous Sums due to the Life Branch	48,985	12	6
	£180,932	6	6

GRESHAM LIFE ASSURANCE SOCIETY.

Established 1848.

GENERAL REPORT OF THE DIRECTORS.

THE Extraordinary General Meeting of the Gresham Life Assurance Society held this day, (14th Nov., 1867,) having empowered the Directors to divide profits at shorter intervals than every five years, the Board of Directors have determined that a division of profits shall be made amongst the Members in respect of surplus funds existing on the 31st July, 1867, and it has been resolved that the future divisions shall take place at intervals of three years, until otherwise determined.

The Directors having instructed their Actuary to make a very careful investigation of the assets and liabilities of the Society as existing on the 31st July, 1867, have now to report thereon.

As regards the Society's assurance and annuity contracts, the ground-work for the valuation of them was the table of mortality known as the Experience of the seventeen offices, and the rate of interest assumed in the valuation 4 per cent. The former being that which, more nearly than any other table, represents the rate of mortality amongst the assured lives in the "Gresham," and the latter being below the average rate of interest actually obtained on the invested funds of the Company, which, being very nearly four-and-a-half per cent. is about one quarter per cent. more than the average rate obtained at the period of the valuation in 1865.

The investigation made by the actuary of the assurance and annuity contracts shows that there were in force on the 31st July, 1867, 19,634 policies for assurance under various contingencies of life. The present value of the sums assured was £3,590,461 19s., and the present value of the prospective income from premiums was £3,395,658 2s.

Also that there were in force at that date 365 policies for annuities, either immediate, contingent, or deferred, the present value of which was £100,723 14s.

From the general Balance-sheet of assets and liabilities appended to this report, it will be seen that the balance in favour of the Company is £665,591 12s. 8d. This amount constitutes the fund, out of which the present bonus and future profits are to be appropriated, and the future expenses are to be paid.

In 1860 and 1865, the policies then existing had been in force $3\frac{1}{2}$ years on the average. The average duration of the policies in force on the 31st July last was rather over 4 years.

The attention of the members was drawn in 1864 to the proportion existing between the realised assets of the Company applicable to the contracts of assurance, and the amount of premiums received on the policies then in force. At that time the proportion was 53 per cent.: it is now 70 per cent.

After a careful consideration of the whole facts, and actuated by prudence and caution on the one hand, and by regard for the interests of the policy-holders on the other, the Directors have determined to apportion amongst the members, by way of bonus, the sum of £60,000.

The Company's income, prospective balance, and realised assets at the periods of the several valuations made of the Company's affairs, are shown in the following tabular statement:—

Valuation. Years.	Income.	Prospective Balances.	Realised Assets.
1852	£23,141	£74,478	£49,662
1855	43,248	105,273	119,377
1860	108,226	206,122	230,166
1865	223,423	312,933	760,796
1867	297,699	665,591	1,025,482

Eighty per cent. of the bonus declared belongs to the participating policy-holders, and will be applicable to all the participating policies in

force on the 31st July last. The bonus applicable to each particular policy will be calculated forthwith so that it may be appropriated on the 30th June next.

In the case of claims under participating policies by death or endowment happening prior to the 30th June, 1868, the cash value of the bonus to be allotted to such policies will be allowed.

In other cases, the usual options will, on the 30th June next, be afforded to the policy-holders in selecting the modes in which the bonus should be applied.

GENERAL BALANCE-SHEET for 31st July, 1867.

ASSETS.		£	s.	d.
Present Value of Policy Premiums, and Reassurances . . .		3,401,463	5	0
Investments in Funded, Freehold and Leasehold Property . . .		456,525	12	9
Mortgages on Real and Personal Estate		340,594	14	0
Loans on Policies on Credit Premiums, etc.		70,527	16	10
Cash, Bills, Bankers' and Agents' Balances, and Current Premiums .		157,833	19	5
Current Interest on Investments		8,318	18	4
		£4,435,264		
			6	4
LIABILITIES.				
Value of Sums Assured and Bonus		3,590,461	19	0
Value of Annuities		100,723	14	0
Sundry Claims and Charges waiting Settlement		56,775	0	8
Proprietors' Fund		21,712	0	0
		£3,769,672		
			13	8
Balance available for present and future Profits and future Expenses		£665,591	12	8
		£4,435,264		
			6	4

NOTICES OF NEW WORKS.

The Insurance Guide and Hand Book: being a Guide to the Principles and Practice of Life Assurance; and a Handbook of the best Authorities on the Science, &c. By CORNELIUS WALFORD, Barrister-at-Law, Fellow of the Statistical Society of Great Britain, &c. &c. Second Edition. C. & E. Layton.

THIS book has established its *raison d'être* in the way of all others most pleasing to an Author. A second Edition is at all times something more than a compliment; in a case like this, it is an evidence of marked, and, we would add, deserved success. A bulky volume of over 400 pages and of proportionate cost, bearing the somewhat forbidding title of "The Insurance Guide and Handbook," and, though dedicated especially to Insurance Agents, having in reality no natural audience, would at first sight seem little likely so completely to exhaust "a first Edition of some Thousands" as to have been "long out of print" ten years after its publication. Yet such is the fact here, and hence this new issue.

On reading the book, it is not difficult to divine the sources of its popularity. The Author himself says, at page 65, "Writing for a popular purpose, and on a subject which certainly *ought* to be popular, we freely